# INFLUENCE OF THE EVAPORATION COEFFICIENT AND TEMPERATURE JUMPS ON THE THERMOPHORESIS OF A MODERATELY LARGE VOLATILE HIGH-VISCOSITY SPHERE IN A BINARY GAS MIXTURE WITH ALLOWANCE FOR VOLUME THERMODIFFUSION AND STEFAN EFFECTS 

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The theory of a uniform thermophoretic motion of a volatile high-viscosity sphere with the phase transition of one component of a moderately rarefied binary gas mixture on its surface has been constructed on the basis of the hydrodynamic method in the regime with slip. The relative influence of the evaporation coefficient and the boundary temperature jumps on the distributions of the velocities, temperatures, and concentrations of the volatile component and the thermophoresis rate has been analyzed. Allowance has been made for the thermodiffusion terms, Stefan effects, and the heat due to the convective transfer of the substance of the condensed phase. The formula obtained has wider limits of application than the existing results. The conclusions of the traditional theories are successfully generalized to the cases of weak and moderately strong processes of diffusion evaporation of a single high-viscosity droplet that moves in a nonuniformly heated binary mixture of gases.

Formulation of the Problem. Let the temperature gradient $\mathbf{A}_{T}$ be created and be kept constant in an unbounded stationary binary gas mixture. A volatile spherical droplet of radius $R$ of a pure liquid with an evaporation coefficient $\alpha$ is placed in such a nonuniformly heated medium (on the surface, we have the phase transition of one component of the gas mixture). Evaporation (condensation) of the molecules of the liquid occurs for Mach numbers much less than unity. The molecules of the condensed phase form the first (volatile) component of the binary gas mixture. The droplet surface is impermeable to the gas molecules of the second (carrier) component. The thermal and diffusion creeps (slips) of the gas along the boundary surface cause the relative motion of a particle in the gaseous medium.

The ordered uniform motion of a particle in the regime with slip is characterized by the thermophoretic velocity $\mathbf{U}_{\mathrm{Tph}}$ in the laboratory coordinate system. The particle is acted upon by thermodiffusion-phoretic and reactive forces $\left(\mathbf{F}_{\mathrm{Tph}}, \mathbf{F}_{\mathrm{Dph}}\right.$, and $\left.\mathbf{F}_{\alpha}\right)$ that are sought to be compensated for with the viscous force of the ambient medium $\mathbf{F}_{\mathbf{v}}$. The thermophoresis rate attains the value $\mathbf{U}_{\mathrm{Tph}}$ if the resultant force disappears, i.e.,

$$
\mathbf{F}=\mathbf{F}_{\mathrm{Tph}}+\mathbf{F}_{\mathrm{Dph}}+\mathbf{F}_{\alpha}+\mathbf{F}_{\mathbf{v}}=0
$$

It is obvious that the influence of volatility on the thermophoresis of a body is bilateral: first, the temperature distribution inside the particle and in the vicinity of it changes, which results in the additional slip of the gaseous medium along the body surface; second, the ambient space becomes saturated with the vapor of a volatile and the thermodiffusion of the components of the gas mixture is enhanced.

The problem is solved in a spherical coordinate system $(r, \theta, \varphi)$. Its origin is rigidly tied to the geometric center of the droplet and the $O z$ axis is directed along the vector $\mathbf{A}_{T}=\left(\nabla T_{\infty}\right)$. Then the particle is at rest in the indicated inertial coordinate system while the center of mass of the external medium is moving with the sought velocity $\mathbf{U}=-\mathbf{U}_{\mathrm{Tph}}$.

[^0]The binary gas mixture is considered to be incompressible, viscous, isotropic, and continuous - the Knudsen number is sufficiently small:

$$
\mathrm{Kn}=\frac{\lambda}{R} \ll 1, \quad \lambda=\max \left(\lambda_{1}, \lambda_{2}\right)
$$

The relative changes in the temperature and concentration of the components of the gas mixture are assumed to be small. This enables us to disregard the temperature and concentration dependences of the coefficients of molecular transfer and to consider them to be constant values for unperturbed values of $T_{0}$ and $C_{0}$. The liberation of heat by internal friction in dissipation of energy is not taken into account.

The dynamics of the droplet occurs for small Reynolds numbers:

$$
\operatorname{Re}=\frac{\rho U R}{\eta} \ll 1
$$

and the nonlinear terms (inertial term and convective ones) are dropped in the equations of slow (creeping) motion of the external medium and of heat and mass transfer. The external mass forces do not act. The motion of the high-viscosity droplet liquid is disregarded. There are no thermal sources outside the aerosol particle and inside it.

Let the times of hydrodynamic, thermal, and concentration relaxations be rather short as compared to the characteristic time of transfer of a droplet. Then the state of an inhomogeneous gaseous medium is described within the framework of hydrodynamic analysis (the macroscopic approach to continua is employed) in the quasistationary approximation (the distributions $\mathbf{v}(\mathbf{r}), p(\mathbf{r}), C(\mathbf{r}), T(\mathbf{r})$, and $T(\mathbf{r})$ are considered to be steady at any instant of time) by the axisymmetric equations of Stokes, continuity, and Laplace:

$$
\eta \Delta \mathbf{v}=\nabla p, \quad \operatorname{div} \mathbf{v}=0, \quad \Delta C=0, \quad \Delta T=0, \quad \Delta T^{\prime}=0
$$

Let the local unit characteristic vectors $\left(\mathbf{n}=\mathbf{i}_{r}, \mathbf{s}=\mathbf{i}_{\theta}, \mathbf{i}_{\varphi}\right)$ form the right-hand system [1]. Then the conditions

$$
\begin{gathered}
r \rightarrow \infty: \mathbf{v}=U \mathbf{i}_{z}, \quad T=T_{0}+A_{T^{z}}, \quad C=C_{0} ; \\
r=R: \mathbf{n}\left(n_{1} \mathbf{v}-\frac{\left(n_{1}+n_{2}\right)^{2} m_{2}}{\rho} D\left(\nabla C+K_{\mathrm{TD}} \nabla \ln T\right)\right)=\alpha v\left(n_{1}+n_{2}\right)\left(C_{\mathrm{s}}-C\right), \\
\mathbf{n}\left(n_{2} \mathbf{v}+\frac{\left(n_{1}+n_{2}\right)^{2} m_{1}}{\rho} D\left(\nabla C+K_{\mathrm{TD}} \nabla \ln T\right)\right)=0, \mathbf{s v}=\mathbf{s}\left(K_{\mathrm{Tsl}}^{\prime} \nabla T+K_{\mathrm{Dsl}}^{\prime} \nabla C\right), \\
T=T^{\prime}+\mathbf{n}\left(V_{T T} \nabla T+V_{T C^{\prime}}^{\prime \prime} \nabla C\right), \mathbf{n}\left(-\kappa \nabla T+\kappa^{\prime} \nabla T^{\prime}\right)=-L m_{1} \alpha v\left(n_{1}+n_{2}\right)\left(C_{\mathrm{s}}-C\right), \\
F_{z}=0, \quad \rho=m_{1} n_{1}+m_{2} n_{2}, \\
C=\frac{n_{1}}{n_{1}+n_{2}}, \quad C_{\mathrm{s}}=\frac{n_{1 \mathrm{~s}}}{n_{1}+n_{2}}, \quad v=\left(\frac{k T}{2 \pi m_{1}}\right)^{1 / 2} .
\end{gathered}
$$

hold at infinity and on the boundary surface.
The above conditions have the following physical meaning:

1. At infinity, the axisymmetric flow of the external medium is uniform in space and has the velocity $U$ in the direction of positive values of the $O z$ axis while the fields of temperature $T(\mathbf{r})$ and relative concentration $C(\mathbf{r})$ of the volatile component of the gas mixture are not perturbed.
2. The normal flow of the first component at the phase boundary is represented as the normal flow of the volatile vapor that is removed from the surface through the Knudsen number and is in proportion to the evaporation


Fig. 1. Rate $U$ of thermophoresis of a droplet of ethyl alcohol vs. evaporation coefficient $\alpha$ for $A_{T}=100 \mathrm{~K} / \mathrm{m}, R=10 \mu \mathrm{~m}, C_{0}=0.01$, and $K_{\mathrm{TD}}=0$ (thermodiffusion is absent). Curves $1 \mathrm{a}-3 \mathrm{a}$ are constructed with allowance for the jump $k_{T T}$ of the temperature in the binary mixture of gases $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}-\mathrm{N}_{2}$, while curves $1 \mathrm{~b}-3 \mathrm{~b}$ are constructed without allowance for the jump. 1a and $1 \mathrm{~b}, 2 \mathrm{a}$ and 2 b , 3 a and 3 b correspond to the unperturbed temperatures $T_{0}=283,303$, and $323 \mathrm{~K} . U, \mu \mathrm{~m} / \mathrm{sec}$.
coefficient $\alpha$. It should be noted that the latter must be replaced by a coefficient of the form $2 \alpha /(2-\alpha)$ that generalizes the Hertz-Knudsen condition [2, 3]. However it is everywhere replaced by $\alpha$ as the main approximation in expansion of the fraction $2 \alpha /(2-\alpha)$ for $\alpha \ll 1$. This is because of the following fact: the numerical analysis shows that, when $\alpha \sim 1$, the rate of thermophoretic transfer of an aerosol particle is independent, in practice, of the coefficient $\alpha$, whereas the indicated relationship is substantial in the case of weak and moderately strong evaporation of the volatile (Fig. 1, where curves a and b reflect the dependence of the thermophoresis rate on the evaporation coefficient for different values of the unperturbed temperature in the presence of the coefficient of temperature jump $k_{T T}$ and in the absence of it respectively).

The surface of the aerosol particle is impermeable to the carrier component. In representing the thermodiffusion force, we drop the barodiffusion term and the term with forces since they do not act on the gas molecules.
3. The tangential component of the external-medium velocity $v_{s}$ is equal to the sum of the rates of thermal and diffusion slips, which are in proportion to the tangential gradients $\nabla_{s} T$ and $\nabla_{s} C$ respectively. The proportionality factors

$$
K_{\mathrm{Tsl}}^{\prime}=K_{\mathrm{Tsl}} \frac{\eta}{\rho T}, \quad K_{\mathrm{Dsl}}^{\prime}=K_{\mathrm{Dsl}} D
$$

are determined by the mathematical methods of the kinetic theory of gases. In the work, we disregard the effect of isothermal slip with a velocity of the order of $O(\mathrm{Kn} U)$.
4. The temperature jumps are caused by the gradients (normal to the interface) of the thermodiffusion quantities in the Knudsen layer. The gaskinetic coefficients $V_{T T}$ and $V_{T C}$ enable us to evaluate the influence of this layer on the vector velocity field $\mathbf{v}(\mathbf{r})$, the scalar distributions $C(\mathbf{r}), T(\mathbf{r})$, and $T(\mathbf{r})$, and the value of the sought rate $U$ of thermophoretic transfer of a moderately large volatile high-viscosity sphere. The coefficients $V_{T T}$ and $V_{T C}$ have a value of the order of $O(\operatorname{KnR})$ and they are found by solution of the system of Boltzmann kinetic equations.
5. The normal heat flux with allowance for plate transitions is continuous at the boundary of the condensed phase. The radial mass flux

$$
\alpha v m_{1}\left(n_{1}+n_{2}\right)\left(C_{\mathrm{s}}(T)-C(T)\right)
$$

of the volatile component of the gas mixture and the difference of the heat flux outside the droplet and inside it are opposite in sign.
6. The resultant force $\mathbf{F}$ acting on the aerosol particle on the source side of the incoming flow of the external medium is equal to zero.

The theory of motion of the aerosol particle disregards a change in the unperturbed temperature $T_{0}$ with time.
In the general case, the partial pressure $p_{1 \mathrm{~s}}$ and the number concentration $n_{1 \mathrm{~s}}$ of a saturated vapor of a pure droplet substance depend on the absolute temperature $T^{\prime}$ of the condensed-phase surface, the electric charge, and the droplet radius. Under the conditions of the problem, the state of the saturated vapor is far from being critical and it is described in the approximation of an ideal gas. We believe that the vapor pressures near the surface with a low curvature and in the vicinity of a plane surface differ insignificantly. Then the approximate integral of the ClausiusClapeyron equation for the dynamic equilibrium of the liquid and gaseous phases of the volatile of an uncharged droplet is written in dimensional form:

$$
p_{1 \mathrm{~s}}(T)=n_{1 \mathrm{~s}}(T) k T=p_{1 \mathrm{~s}}\left(T_{\mathrm{w}}\right) \exp \left\{\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}\left(1-\frac{T_{\mathrm{w}}}{T}\right)\right\}
$$

The temperature dependence of the relative concentration of the saturated vapor of the volatile

$$
C_{\mathrm{s}}(T)=\frac{n_{1 \mathrm{~s}}(T)}{n_{1}+n_{2}}=C_{\mathrm{s}}\left(T_{\mathrm{w}}\right) \frac{T_{\mathrm{w}}}{T} \exp \left\{\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}\left(1-\frac{T_{\mathrm{w}}}{T}\right)\right\}
$$

is expanded in a Taylor series in powers $\left(T-T_{\mathrm{w}}\right)$ :

$$
C_{\mathrm{s}}(T)=C_{\mathrm{s}}\left(T_{\mathrm{w}}\right)+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=T_{\mathrm{w}}}\left(T-T_{\mathrm{w}}\right)+\ldots .
$$

In the case of a low nonisothermicity of the droplet, we retain, in the expansion, the first two terms:

$$
\begin{gathered}
\frac{T_{\mathrm{w}}}{T}=1-\frac{T-T_{\mathrm{w}}}{T_{\mathrm{w}}}+\ldots, \exp \left\{\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}\left(1-\frac{T_{\mathrm{w}}}{T}\right)\right\}=1+\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}} \frac{T-T_{\mathrm{w}}}{T_{\mathrm{w}}}-\ldots, \\
\frac{T_{\mathrm{w}}}{T} \exp \left\{\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}\left(1-\frac{T_{\mathrm{w}}}{T}\right)\right\}=1+\left(\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}-1\right) \frac{T-T_{\mathrm{w}}}{T_{\mathrm{w}}}+\ldots, \\
C_{\mathrm{s}}(T)=C_{\mathrm{s}}\left(T_{\mathrm{w}}\right)\left\{1+\left(\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}-1\right) \frac{T-T_{\mathrm{w}}}{T_{\mathrm{w}}}+\ldots\right\} .
\end{gathered}
$$

Therefore, we have the estimate of the temperature change of the function $C_{\mathrm{S}}(T)$ at the point $T=T_{\mathrm{w}}$ :

$$
\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=T_{\mathrm{w}}}=\frac{1}{T_{\mathrm{w}}} C_{\mathrm{s}}\left(T_{\mathrm{w}}\right)\left(\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}-1\right), \quad C_{\mathrm{s}}\left(T_{\mathrm{w}}\right)=C_{\mathrm{s}}\left(T_{0}\right) \frac{T_{0}}{T_{\mathrm{w}}} \exp \left\{\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}\left(\frac{T_{\mathrm{w}}}{T_{0}}-1\right)\right\},
$$

since the equality

$$
C_{\mathrm{s}}\left(T_{0}\right)=C_{\mathrm{s}}\left(T_{\mathrm{w}}\right) \frac{T_{\mathrm{w}}}{T_{0}} \exp \left\{\frac{L \mu}{R_{\mathrm{g}} T_{\mathrm{w}}}\left(1-\frac{T_{\mathrm{w}}}{T_{0}}\right)\right\}
$$

holds.
Thus, the linearized boundary conditions have the form

$$
v_{\theta}=K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0} T_{0}} \nabla_{\theta} T+K_{\mathrm{Dsl}} D \nabla_{\theta} C
$$

$$
\begin{gathered}
n_{10} v_{r}-\frac{n_{0}^{2} m_{2}}{\rho_{0}} D\left(\nabla_{r} C+\frac{K_{\mathrm{TD}}}{T_{0}} \nabla_{r} T\right)=\alpha v n_{0}\left\{C_{\mathrm{s}}\left(T_{\mathrm{w}}\right)+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=T_{\mathrm{w}}}\left(T^{\prime}-T_{\mathrm{w}}\right)-C\right\}, \\
n_{20} v_{r}+\frac{n_{0}^{2} m_{1}}{\rho_{0}} D\left(\nabla_{r} C+\frac{K_{\mathrm{TD}}}{T_{0}} \nabla_{r} T\right)=0, \quad T=T^{\prime}+V_{T T} \nabla_{r} T+V_{T C} T^{\prime} \nabla_{r} C \\
-\kappa_{0} \nabla_{r} T+\kappa_{0}^{\prime} \nabla_{r} T^{\prime}=-L m_{1} \alpha v n_{0}\left\{C_{\mathrm{s}}\left(T_{\mathrm{w}}\right)+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=T_{\mathrm{w}}}\left(T^{\prime}-T_{\mathrm{w}}\right)-C\right\}, F_{z}=0, \\
n_{0}=n_{10}+n_{20}, \quad \rho_{0}=m_{1} n_{10}+m_{2} n_{20}, \quad \eta_{0}=\eta\left(T_{0}, C_{0}\right), \quad \kappa_{0}=\kappa\left(T_{0}, C_{0}\right), \quad \kappa_{0}^{\prime}=\kappa^{\prime}\left(T_{0}\right) .
\end{gathered}
$$

The solution of the axisymmetric hydrodynamic problem for an incompressible gas medium can conveniently be represented in terms of the stream function $\Psi(\mathbf{r})$. In the right-hand system of orthogonal spherical coordinates $(r$, $\theta, \varphi$ ), the velocity components and the $z$ projection of the resultant force acting on the sphere $r=R$ in the Stokes approximation are determined in terms of the stream function $\Psi=\Psi(r, \theta)$ from the formulas [1]

$$
v_{r}=-\frac{1}{r^{2} \sin \theta} \frac{\partial \Psi}{\partial \theta}, \quad v_{\theta}=\frac{1}{r \sin \theta} \frac{\partial \Psi}{\partial r}, \quad v_{\varphi}=0, \quad F_{z}=\pi \eta_{0} \int_{0}^{\pi} r^{4} \sin \theta \frac{\partial}{\partial r}\left\{\frac{E^{2} \Psi}{r^{2}}\right\} d \theta .
$$

Upon making the physical quantities in the equations of hydrodynamics and heat and mass transfer and in the boundary conditions dimensionless

$$
\tilde{r}=\frac{r}{R}, \quad \tilde{v}_{r}=\frac{v_{r}}{U}, \quad \tilde{v}_{\theta}=\frac{v_{\theta}}{U}, \quad \tilde{\Psi}=\frac{\Psi}{U R^{2}}, \quad \tilde{T}=\frac{T-T_{0}}{A_{T} R}, \quad \tilde{T}^{\prime}=\frac{T^{\prime}-T_{0}}{A_{T} R}, \quad \tilde{F}_{z}=\frac{F_{z}}{6 \pi \eta_{0} R U}
$$

(the tilde above is dropped), the formulation of the problem has the following linearized form:

$$
\begin{gather*}
\mathrm{E}^{4} \Psi(r, \xi)=0, \Delta T=0, \Delta T=0, \Delta C=0 ;  \tag{1}\\
r \rightarrow \infty: \Psi(r, \xi)=-\frac{1}{2} r^{2}\left(1-\xi^{2}\right), T=z, C=C_{0} ;  \tag{2}\\
r=1:\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U v_{r}=\alpha v\left\{C_{\mathrm{s}}(\tau)+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}(T-\tau)-C\right\} ;  \tag{3}\\
\left(1-C_{0}\right)\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U v_{r}+\frac{D}{R}\left\{\frac{\partial C}{\partial r}+K_{\mathrm{TD}} \varepsilon \frac{\partial T}{\partial r}\right\}=0 ;  \tag{4}\\
U v_{\theta}=-K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0} T_{0}} A_{T} \sqrt{1-\xi^{2}} \frac{\partial T}{\partial \xi}-K_{\mathrm{Dsl}} \frac{D}{R} \sqrt{1-\xi^{2}} \frac{\partial C}{\partial \xi}  \tag{5}\\
\varepsilon T=\varepsilon T^{\prime}+\varepsilon k_{T T} \frac{\partial T}{\partial r}+k_{T C} \frac{\partial C}{\partial r} ; \tag{6}
\end{gather*}
$$

$$
\begin{gather*}
-\frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial T}{\partial r}+\frac{\partial T^{\prime}}{\partial r}=-\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}\left\{C_{\mathrm{s}}(\tau)+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\left(T^{\prime}-\tau\right)-C\right\} ;  \tag{7}\\
F_{z}=0, \tag{8}
\end{gather*}
$$

where

$$
\tau=\frac{T_{\mathrm{w}}-T_{0}}{A_{T} R} \ll 1 ; \quad \varepsilon=\frac{A_{T} R}{T_{0}} \ll 1 ; \quad k_{T T}=\frac{V_{T T}}{R} \sim O(\mathrm{Kn}) ; k_{T C}=\frac{V_{T C}}{R} \sim O(\mathrm{Kn}) .
$$

The differential operators of Stokes and Laplace in the spherical coordinate system have the structure ( $\mathrm{E}^{4}=$ $\left.E^{2}\left(E^{2}\right)\right)$

$$
\mathrm{E}^{2}=\frac{\partial^{2}}{\partial r^{2}}+\frac{1-\xi^{2}}{r^{2}} \frac{\partial^{2}}{\partial \xi^{2}}, \quad \Delta=\frac{1}{r^{2}} \frac{\partial}{\partial r}\left\{r^{2} \frac{\partial}{\partial r}\right\}+\frac{1}{r^{2}} \frac{\partial}{\partial \xi}\left\{\left(1-\xi^{2}\right) \frac{\partial}{\partial \xi}\right\},-1 \leq \xi=\cos \theta \leq+1 .
$$

If a droplet of a pure liquid is coated with a thin film of any contaminant, such an aerosol particle is formally considered as being nonvolatile (evaporation coefficient is $\alpha \rightarrow 0$ ). In this case, the condition of impermeability of the condensed-phase surface to the external medium holds, i.e., the normal component of the velocity at the boundary $r=1$ vanishes.

Determination of the Rate of Thermophoretic Transfer of a Particle. The solutions of Eqs. (1) in general form are represented as

$$
\begin{gathered}
\Psi(r, \xi)=\sum_{n=2}^{\infty}\left\{a_{n} r^{n}+b_{n} r^{-n+1}+c_{n} r^{n+2}+d_{n} r^{-n+3}\right\} J_{n}(\xi), \\
v_{r}(r, \xi)=-\sum_{n=2}^{\infty}\left\{a_{n} r^{n-2}+b_{n} r^{-n-1}+c_{n} r^{n}+d_{n} r^{-n+1}\right\} P_{n-1}(\xi), \\
v_{\theta}(r, \xi)=\sum_{n=2}^{\infty}\left\{n a_{n} r^{n-2}-(n-1) b_{n} r^{-n-1}+(n+2) c_{n} r^{n}-(n-3) d_{n} r^{-n+1}\right\} \frac{J_{n}(\xi)}{\sqrt{1-\xi^{2}}}, \\
\frac{T}{T}(r, \xi)=\sum_{n=0}^{\infty}\left\{\frac{e_{n}}{e_{n}} r^{n}+\frac{f_{n}}{f_{n}^{\prime}} r^{-n-1}\right\} P_{n}(\xi), C(r, \xi)=\sum_{n=0}^{\infty}\left\{g_{n} r^{n}+h_{n} r^{-n-1}\right\} P_{n}(\xi) .
\end{gathered}
$$

From conditions (2), with allowance for the finiteness of the temperature at the center of the droplet, we have

$$
\begin{aligned}
& a_{2}=-1, a_{n}=0, \text { if } n \geq 3 ; c_{n}=0, \text { if } n \geq 2 ; e_{0}=0, e_{1}=1, e_{n}=0, \text { if } n \geq 2 ; \\
& f_{n}^{\prime}=0, \text { if } n \geq 0 ; g_{0}=C_{0}, g_{n}=0, \text { if } n \geq 1 .
\end{aligned}
$$

As a result, we obtain the expansions

$$
\Psi(r, \xi)=-r^{2} J_{2}(\xi)+\sum_{n=2}^{\infty}\left\{b_{n} r^{-n+1}+d_{n} r^{-n+3}\right\} J_{n}(\xi), \quad T(r, \xi)=r \xi+\sum_{n=0}^{\infty} f_{n} r^{-n-1} P_{n}(\xi),
$$

$$
T^{\prime}(r, \xi)=\sum_{n=0}^{\infty} e_{n}^{\prime} r^{n} P_{n}(\xi), \quad C(r, \xi)=C_{0}+\sum_{n=0}^{\infty} h_{n} r^{-n-1} P_{n}(\xi)
$$

which are substituted into boundary conditions (3)-(7). Then, using properties (A1)-(A3) of ultraspherical Gegenbauer polynomials of order $n$ and degree $\pm \frac{1}{2}$

$$
J_{n}(\xi)=C_{n}^{-1 / 2}(\xi), \quad P_{n}(\xi)=C_{n}^{+1 / 2}(\xi)
$$

by virtue of orthogonality conditions of the (A5)-(A6) type, for integration constants we can write the algebraic equations

$$
\begin{align*}
& C_{\mathrm{s}}(\tau)-C_{0}+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\left(e_{0}^{\prime}-\tau\right)-h_{0}=0,  \tag{9}\\
& -\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U\left(-1+b_{2}+d_{2}\right)=\alpha v\left\{\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau} e_{1}^{\prime}-h_{1}\right\},  \tag{10}\\
& -\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U\left(b_{n+1}+d_{n+1}\right)=\alpha \nu\left\{\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau} e_{n}^{\prime}-h_{n}\right\},  \tag{11}\\
& h_{0}+\varepsilon K_{\mathrm{TD}} f_{0}=0,  \tag{12}\\
& \left(1-C_{0}\right)\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U\left(-1+b_{2}+d_{2}\right)+\frac{D}{R}\left\{2 h_{1}+\varepsilon K_{\mathrm{TD}}\left(-1+2 f_{1}\right)\right\}=0,  \tag{13}\\
& \left(1-C_{0}\right)\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U\left(b_{n+1}+d_{n+1}\right)+\frac{D}{R}(n+1)\left\{h_{n}+\varepsilon K_{\mathrm{TD}} f_{n}\right\}=0,  \tag{14}\\
& U\left(2+b_{2}-d_{2}\right)=2 K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0} T_{0}} A_{T}\left(1+f_{1}\right)+2 K_{\mathrm{Dsl}} \frac{D}{R} h_{1},  \tag{15}\\
& U\left\{n b_{n+1}+(n-2) d_{n+1}\right\}=K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0} T_{0}} A_{T} n(n+1) f_{n}+K_{\mathrm{Dsl}} \frac{D}{R} n(n+1) h_{n},  \tag{16}\\
& \varepsilon\left(1+k_{T T}\right) f_{0}-\varepsilon e_{0}^{\prime}+k_{T C} h_{0}=0,  \tag{17}\\
& \varepsilon\left(1+2 k_{T T}\right)\left(1+f_{1}\right)-\varepsilon e_{1}^{\prime}+2 k_{T C} h_{1}=3 \varepsilon k_{T T},  \tag{18}\\
& \varepsilon\left\{1+(n+1) k_{T T}\right\} f_{n}-\varepsilon e_{n}^{\prime}+(n+1) k_{T C} h_{n}=0, \tag{19}
\end{align*}
$$

$$
\begin{align*}
& \frac{\kappa_{0}}{\kappa_{0}^{\prime}} f_{0}=-\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}\left\{C_{\mathrm{s}}(\tau)-C_{0}+\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\left(e_{0}^{\prime}-\tau\right)-h_{0}\right\},  \tag{20}\\
& \frac{\kappa_{0}}{\kappa_{0}^{\prime}}\left(-1+2 f_{1}\right)+e_{1}^{\prime}=-\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}\left\{\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau} e_{1}^{\prime}-h_{1}\right\},  \tag{21}\\
& \frac{\kappa_{0}}{\kappa_{0}^{\prime}}(n+1) f_{n}+n e_{n}^{\prime}=-\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}\left\{\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau} e_{n}^{\prime}-h_{n}\right\}, \tag{22}
\end{align*}
$$

where $n \geq 2$.
From relations (9), (12), (17), and (20) we find

$$
\begin{equation*}
f_{0}=e_{0}^{\prime}=h_{0}=0, \quad C_{\mathrm{s}}(\tau)-C_{0}-\left.\tau \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}=0 . \tag{23}
\end{equation*}
$$

Integration over the surface of the sphere $r=1$ with account for relations (A4) yields

$$
F_{z}=\frac{1}{6} \int_{-1}^{+1} r^{4} \frac{\partial}{\partial r}\left\{\frac{E^{2} \Psi}{r^{2}}\right\} d \xi=\frac{2}{3} d_{2}
$$

whence the coefficient $d_{2}$ vanishes by virtue of condition (8).
From Eqs. (10) and (15) with account for equality (18) we obtain

$$
\begin{gathered}
3\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} U=-\left.3 k_{T T} \alpha v \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}+ \\
+\left[2\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0} T_{0}} A_{T}+\left.\alpha v\left(1+2 k_{T T}\right) \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right]\left(1+f_{1}\right)+ \\
+\left[2\left\{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}\right\} K_{\mathrm{Dsl}} \frac{D}{R}-\alpha v\left(1-\left.2 \frac{k_{T C}}{\varepsilon} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)\right] h_{1} .
\end{gathered}
$$

By virtue of relations (10), (13), and (18) we find

$$
\begin{gathered}
\left\{\left.\left(1-C_{0}\right)\left(1+2 k_{T T}\right) \frac{\alpha v R}{D} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}-2 \varepsilon K_{\mathrm{TD}}\right\}\left(1+f_{1}\right)- \\
-\left[2+\left(1-C_{0}\right) \frac{\alpha \vee R}{D}\left(1-\left.2 \frac{k_{T C}}{\varepsilon} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)\right] h_{1}=3\left\{\left.\left(1-C_{0}\right) k_{T T} \frac{\alpha v R}{D} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}-\varepsilon K_{\mathrm{TD}}\right\} .
\end{gathered}
$$

Equation (21) with account for equality (18) is written in the form

$$
\left[2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}}+\left(1+2 k_{T T}\right)\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)\right]\left(1+f_{1}\right)-
$$

$$
-\left[\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}-2 \frac{k_{T C}}{\varepsilon}\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)\right] h_{1}=3\left[\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)\right] .
$$

Solution of the system of the last two equations yields $1+f_{1}=3 \delta^{\prime} / \delta$ and $h_{1}=3 \delta^{\prime \prime} / \delta$,

$$
\begin{gather*}
\delta=2\left(1-C_{0}\right) \frac{\alpha \nu R}{D}\left\{k_{T T}-\left.2 \frac{k_{T C}}{\varepsilon} \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right\}+4\left(k_{T T}-k_{T C} K_{\mathrm{TD}}\right)\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)+ \\
+\left(1+2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}}\right)\left\{2+\left(1-C_{0}\right) \frac{\alpha v R}{D}\right\}+2 \frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}\left\{\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}+\varepsilon K_{\mathrm{TD}}\right\},  \tag{24}\\
\delta^{\delta^{\prime}=\left(1-C_{0}\right) \frac{\alpha v R}{D}\left\{k_{T T}-\left.2 \frac{k_{T C}}{\varepsilon} \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right\}+2\left(k_{T T}-k_{T C} K_{\mathrm{TD}}\right)\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)+} \\
\quad+\frac{\kappa_{0}}{\kappa_{0}^{\prime}}\left\{2+\left(1-C_{0}\right) \frac{\alpha v R}{D}\right\}+\varepsilon K_{\mathrm{TD}} \frac{L m_{1} \alpha v n_{0}}{A_{T}{\kappa_{0}^{\prime}}^{\partial}},  \tag{25}\\
\delta^{\prime \prime}=\left.\frac{\kappa_{0}}{\kappa_{0}^{\prime}}\left(1-C_{0}\right) \frac{\alpha v R}{D} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}+\varepsilon K_{\mathrm{TD}}\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right) . \tag{26}
\end{gather*}
$$

It is obvious that the relation

$$
\delta=2 \delta^{\prime}+\left(1-C_{0}\right) \frac{\alpha v R}{D}+2\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)
$$

holds.
Thus, we obtain the expression for the velocity $\mathbf{U}=-\mathbf{U}_{\mathrm{Tph}}$ of the center of inertia of the gaseous medium relative to the droplet:

$$
\begin{equation*}
U=2 K_{\mathrm{Ts} 1} \frac{\eta_{0}}{\rho_{0} T_{0}} A_{T} \frac{\delta^{\prime}}{\delta}+2 K_{\mathrm{Ds} 1} \frac{D}{R} \frac{\delta^{\prime \prime}}{\delta}+\frac{\alpha \nu}{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}}\left(\left.2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}-\varepsilon K_{\mathrm{TD}}\right) \frac{1}{\delta} \tag{27}
\end{equation*}
$$

Equations (11), (14), (16), (19), and (22) yield

$$
b_{n+1}=d_{n+1}=f_{n}=e_{n}^{\prime}=h_{n}=0, \quad n \geq 2 .
$$

By solution of the thermodiffusion problem we obtain that the average reduced temperature $\tau$ on the boundary surface is equal to zero. Then the transcendental algebraic equation (23) shows that the theory constructed holds for $C_{0} \rightarrow C_{\mathrm{s}}\left(T_{0}\right)$ when the binary mixture of gases is sufficiently saturated with the vapor of the volatile component. The diffusion regime of evaporation of the high-viscosity sphere is observed, as a rule, for ordinary temperatures, when $C_{\mathrm{S}}\left(T_{0}\right) \ll 1$.

Analysis of the Results. In formula (27), the first and second terms proportional to the gaskinetic coefficients $K_{\text {Tsl }}$ and $K_{\text {Dsl }}$ are determined by, respectively, the thermal and diffusion slips of the binary gas mixture. The third term describes the reactive part of a momentum that acts on the aerosol particle and is related to the phase transition on its surface. The temperature dependence of the relative concentration of the saturated vapor of the volatile droplet
substance and the thermodiffusion phenomena in the volume of the gas mixture cause a nonuniform evaporation (condensation) on the condensed-phase surface. As a result, we observe the reactive effect.

The derivation of the traditional theory [4] of thermophoresis of a volatile one-component spherical droplet in a moderately rarefied binary gas mixture in the limiting case of a high-viscosity sphere in the absence of concentration jumps, thermodiffusion and Barnett terms, and the Dufour effect has the error committed in representation of the reactive component of the velocity $U$. The calculation formula has been obtained with allowance for the heat flux on the particle surface, which is due to the convective transfer of the volatile mass, and it must have the form

$$
\begin{aligned}
& U=\frac{2}{\delta}\left\{K_{\mathrm{Tsl}}^{\prime} A_{T} \delta^{\prime}+\left.K_{\mathrm{Dsl}} \frac{D}{R} \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}+\left.\frac{1}{1-C_{0}}\left(\frac{1}{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}}+2 \frac{n_{0}}{n_{0}^{\prime}}\right) \frac{D}{R} \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right\}, \\
& \delta=1+2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}}+\left.\frac{2}{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}} \frac{L m_{2} n_{0} D}{A_{T} R \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}+ \\
& +2 k_{T T}\left(1+\left.\frac{2}{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}} \frac{L m_{2} n_{0} D}{A_{T} R \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)-\left.4 k_{T C} \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{T_{0}}{A_{T} R} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}, \\
& \delta^{\prime}=\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}\left(1+\left.\frac{2}{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}} \frac{L m_{2} n_{0} D}{A_{T} R \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)-\left.2 k_{T C} \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{T_{0}}{A_{T} R} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau} .
\end{aligned}
$$

This result completely coincides with formula (27) in the absence of thermodiffusion and in the case of a rather strong diffusion evaporation of the volatile substance of the particle where

$$
C_{0} \ll 1 \ll \frac{\alpha v R}{D}, \frac{n_{0}}{n_{0}^{\prime}} \ll 1 .
$$

The evaporation (condensation) coefficient is usually defined as the probability that a vapor molecule upon its arrival at the liquid surface will not reflect from it. The value of the coefficient is found in an indirect manner, for example, in the process of measurements of the rate of condensation growth or evaporation of droplets. It should be noted that experimental determination of $\alpha$ is a difficult problem at present and the data given in the literature on this subject are contradictory. Thus, a detailed analysis of numerous procedures of measurement of the coefficient $\alpha$ [5, 6] has shown that experimental data [7, 8] for water droplets contain low values of $\alpha \sim 0.01$, whereas in other experimental investigations [9,10] a value of $\alpha \sim 1$ has been obtained.

Formula (27) yields that for

$$
\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}>0, \quad K_{\mathrm{TD}}=k_{T T}=k_{T C}=0
$$

the particle seeks to move to colder regions of the gas medium ("positive" factors) due to the first and third terms of it. The action of the contribution of the second term on the direction of the thermophoresis rate depends on the sign of the coefficient $K_{\mathrm{Dsl}}$ : the rate is directed toward the vector $\mathbf{A}_{T}$ when $m_{1}>m_{2}\left(K_{\mathrm{Dsl}}<0\right)$, and its direction is opposite when $m_{1}<m_{2}\left(K_{\mathrm{Dsl}}>0\right)$.

Analysis of the equations for $\left(1+f_{1}\right)$ and $h_{1}$ shows that one can disregard the influence of volatility on the thermodiffusion scalar fields and the velocity of thermophoretic motion of a particle when: 1) the relative temperature differences are larger than the change in the relative concentration of the saturated vapor of the volatile component of
the gas mixture; 2) diffusion evaporation of the low-thermal-conductivity substance of a droplet is weak; 3) the action of thermodiffusion effects is quite substantial:

$$
\frac{\alpha \nu R}{D} \ll 1,\left.\quad \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau} \sim \varepsilon\left|K_{\mathrm{TD}}\right|, \quad k_{T C} K_{\mathrm{TD}} \sim 1, \quad \frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \ll \frac{k_{T C}}{\varepsilon}
$$

The temperature field outside the droplet and inside it and the distribution of the relative concentration of the volatile component of the gas mixture

$$
\begin{aligned}
& T(r, \xi)=r \xi+f_{1} r^{-2} \xi, \quad T^{\prime}(r, \xi)=e_{1}^{\prime} r \xi, \quad C(r, \xi)=C_{0}+h_{1} r^{-2} \xi, f_{1}=3 \frac{\delta^{\prime}}{\delta}-1, e_{1}^{\prime}=3 \frac{\delta^{\prime}}{\delta} \\
& h_{1}=3 \frac{\delta^{\prime \prime}}{\delta}, \delta \rightarrow 2\left\{1+2\left(\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}-k_{T C} K_{\mathrm{TD}}\right)\right\}, \quad \delta^{\prime} \rightarrow 2\left(\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}-k_{T C} K_{\mathrm{TD}}\right), \quad \delta^{\prime \prime} \rightarrow \varepsilon K_{\mathrm{TD}}
\end{aligned}
$$

correspond to the solid nonvolatile aerosol particle. Then, with allowance for the boundary temperature jumps and thermodiffusion effects, the value of the rate of thermophoretic transfer is equal to

$$
U^{\prime}=K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0} T_{0}} A_{T} \frac{2\left(\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}-k_{T C} K_{\mathrm{TD}}\right)}{1+2\left(\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}-k_{T C} K_{\mathrm{TD}}\right)}+K_{\mathrm{Dsl}} \frac{D}{T_{0}} A_{T} \frac{K_{\mathrm{TD}}}{1+2\left(\frac{\kappa_{0}}{\kappa_{0}^{\prime}}+k_{T T}-k_{T C} K_{\mathrm{TD}}\right)}
$$

Upon combining the above conditions and the formula

$$
\left.\frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}=\varepsilon C_{\mathrm{S}}\left(T_{0}\right)\left(\frac{L \mu}{R_{\mathrm{g}} T_{0}}-1\right)
$$

we have the following criteria enabling us to disregard the "volatility" of the droplet:

$$
\frac{\alpha v R}{D} \ll 1, \quad C_{\mathrm{s}}\left(T_{0}\right)\left(\frac{L \mu}{R_{\mathrm{g}} T_{0}}-1\right) \sim\left|K_{\mathrm{TD}}\right|, \quad k_{T C} K_{\mathrm{TD}} \sim 1, \quad \frac{L m_{1} \alpha \nu R n_{0}}{\kappa_{0}^{\prime}} \ll k_{T C} T_{0}
$$

The distributions of the temperature and the relative concentration of the component are independent, in practice, of the thermal conductivity of the gas medium if the droplet is of high thermal conductivity. Around the volatile particle, there is formed a gas mixture which transfers an uncompensated momentum to the body mainly by thermodiffusion:

$$
\begin{gathered}
\delta \rightarrow\left(1-C_{0}\right) \frac{\alpha v R}{D}\left(1+2 k_{T T}\right)+2 \varepsilon K_{\mathrm{TD}} \frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}+2\left(1+2\left(k_{T T}-k_{T C} K_{\mathrm{TD}}\right)\right)\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right), \\
\delta^{\prime} \rightarrow\left(1-C_{0}\right) \frac{\alpha v R}{D} k_{T T}+\varepsilon K_{\mathrm{TD}} \frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}}+2\left(k_{T T}-k_{T C} K_{\mathrm{TD}}\right)\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T} \kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right) \\
\delta^{\prime \prime} \rightarrow \varepsilon K_{\mathrm{TD}}\left(1+\left.\frac{L m_{1} \alpha v n_{0}}{A_{T K_{0}^{\prime}}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}\right)
\end{gathered}
$$



Fig. 2. Rate $U$ of transfer of a pure droplet of ethyl alcohol vs. its radius $R$. Curve 1 is constructed according to the formula of [4], while curves 2 and 3 employ expressions (24)-(27) for $\alpha=1$ and $\alpha=0$ respectively ( $T_{0}=323 \mathrm{~K}$, $\left.C_{0}=0.1, k_{T C}=0, K_{\mathrm{TD}}=0, A_{T}=100 \mathrm{~K} / \mathrm{m}\right) . U, \mu \mathrm{~m} / \mathrm{sec} ; R, \mu \mathrm{~m}$.

It is of interest to investigate the dependence $U=U(\alpha)$. After intermediate computations we obtain

$$
\begin{gathered}
\frac{d U}{d \alpha}=2 \nu \Phi\left(\kappa_{0}, \kappa_{0}^{\prime}\right)\left\{\left.2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}-\varepsilon K_{\mathrm{TD}}\right\} \frac{1}{\delta^{2}}, \\
\Phi\left(\kappa_{0}, \kappa_{0}^{\prime}\right)=\frac{1}{C_{0}+\left(1-C_{0}\right) \frac{m_{2}}{m_{1}}}\left(1+2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}}+2\left(k_{T T}-k_{T C} K_{\mathrm{TD}}\right)\right)- \\
-2 K_{\mathrm{Tsl}} \frac{\eta_{0}}{\rho_{0}}\left(\left(1-C_{0}\right) \frac{k_{T C}}{D}+\frac{L m_{1} n_{0}}{T_{0} \kappa_{0}^{\prime}}\right)+K_{\mathrm{Dsl}}\left(\left(1-C_{0}\right)\left(1+2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}}+2 k_{T T}\right)+2 K_{\mathrm{TD}} \frac{L m_{1} n_{0} D}{T_{0} \kappa_{0}^{\prime}}\right) .
\end{gathered}
$$

The dependence $U(\alpha)$ has a monotone character (decreasing or increasing). The quantity $\mathbf{U}_{\text {Tph }}$ is not related to the quantity $\alpha$ in two cases where the derivative of the function $U=U(\alpha)$ vanishes:

$$
\Phi\left(\kappa_{0}, \kappa_{0}^{\prime}\right)=0,\left.\quad 2 \frac{\kappa_{0}}{\kappa_{0}^{\prime}} \frac{\partial C_{\mathrm{s}}}{\partial T}\right|_{T=\tau}-\varepsilon K_{\mathrm{TD}}=0
$$

The first relation between the physical quantities which characterizes the state of media outside the high-viscosity droplet and inside it potentially holds for low-thermal-conductivity aerosol particles, while the second equality holds for high-thermal-conductivity particles.

A numerical analysis for a large $(R=100 \mu \mathrm{~m})$ particle and a moderately large ( $R=10 \mu \mathrm{~m}$ ) particle shows that in weak evaporation of a droplet of ethyl alcohol in the binary mixture of gases $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}-\mathrm{N}_{2}$, the thermophoresis rate depends very strongly on the coefficient $\alpha$. The above relationship is very weak if $0.05<\alpha<1$. At ordinary temperatures, the volatility of the alcohol increases the velocity of thermophoretic motion by $10-13 \%$ as compared to the velocity $U^{\prime}\left(C_{0}=0.001-0.1, k_{T C}=0, K_{\mathrm{TD}}=0\right)$. This conclusion is in agreement with the results of [11, 12]. However the conclusions in these investigations devoted to solution of the problem of thermophoresis of a volatile spherical aerosol particle repeat, in fact, results of the works references to which have been in given in [13, 14] but which have not been presented in sufficient detail in [11, 12].

The influence of a temperature jump for moderately large particles ( $R=10 \mu \mathrm{~m}$ ) leads to an increase of $6-8 \%$ in the thermophoresis rate, which is clearly seen in Fig. 1.

This work represent an alternative to the existing traditional theories [4, 15-17]. Its results coincide with those obtained earlier only when the condition $C_{\mathrm{s}}\left(T_{0}\right) \ll 1$ holds. However, in passage to a moderately large volatile pure high-viscosity aerosol particle the inequality $\alpha v R / D \gg 1$ fails even in very strong diffusion evaporation, when $\alpha \sim 1$. In other words, the rate of thermophoretic transfer of a single moderately large volatile solid sphere in the regime of
weak and moderately strong processes of diffusion evaporation $(\alpha<0.1)$ has dissimilar values if it is computed using expressions (24)-(27) and the formula from [4]. Figure 2 gives the dependences of the value of the rate $U$ of thermophoresis on the radius $R$ of a spherical droplet of ethyl alcohol in the mixture of gases $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}-\mathrm{N}_{2}$; they demonstrate that in strong evaporation of a large aerosol particle $(\alpha \rightarrow 1)$, the disagreement between the results of this work and the traditional theory [4] attains $15 \%$. This difference increases to $30 \%$ if the binary gas mixture is saturated with the vapor of the volatile component when $C_{0} \rightarrow C_{\mathrm{s}}\left(T_{0}\right)$, all other things being equal. The theory constructed in the present work is, probably, more general.

## APPENDIX

$$
\begin{align*}
& J_{n}(\xi)=\frac{P_{n-2}(\xi)-P_{n}(\xi)}{2 n-1}, n \geq 2 ;  \tag{A1}\\
& \frac{d J_{n}(\xi)}{d \xi}=-P_{n-1}(\xi), \quad n \geq 1 ;  \tag{A2}\\
& \left(1-\xi^{2}\right) \frac{d P_{n}(\xi)}{d \xi}=n(n+1) J_{n+1}(\xi), n \geq 0 ;  \tag{A3}\\
& \int_{-1}^{+1} J_{n}(\xi) d \xi= \begin{cases}2, & 0, \\
\frac{2}{3}, & 2, \\
0, & n \neq 0,2 ;\end{cases}  \tag{A4}\\
& \int_{-1}^{+1} \frac{J_{m}(\xi) J_{n}(\xi)}{1-\xi^{2}} d \xi= \begin{cases}0, & m \neq n, \\
\frac{2}{n(n-1)(2 n-1)}, & m=n ;\end{cases}  \tag{A5}\\
& \int_{-1}^{+1} P_{m}(\xi) P_{n}(\xi) d \xi= \begin{cases}0, & m \neq n, \\
\frac{2}{2 n+1}, & m=n .\end{cases} \tag{A6}
\end{align*}
$$

Orthogonality conditions of the (A5)-(A6) type hold, when $m \neq 0$ and 1 and $n \neq 0$ and 1 .

## NOTATION

$\mathbf{r}$, radius vector of the point of observation, $\mathrm{m} ;(r$ and $\theta, \varphi)$, spherical coordinates, m and $\mathrm{rad} ;\left(\mathbf{i}_{r}, \mathbf{i}_{\theta}, \mathbf{i}_{\varphi}\right)$, dimensionless unit vectors corresponding to them; $\left(\mathbf{n}, \mathbf{s}, \mathbf{i}_{\varphi}\right)$, right-hand set of three dimensionless local characteristic unit vectors; $R$, radius of curvature of the spherical surface of a high-viscosity droplet, $\mathrm{m} ; \mathbf{U}$, velocity of the center of inertia of the binary gas mixture, $\mathrm{m} / \mathrm{sec} ; \mathbf{U}_{\mathrm{Tph}}=-\mathbf{U}$, velocity of uniform thermophoretic motion of a volatile particle, $\mathrm{m} / \mathrm{sec}$; $\mathbf{F}_{\mathrm{Tph}}, \mathbf{F}_{\mathrm{Dph}}, \mathbf{F}_{\alpha}, \mathbf{F}_{\mathbf{v}}$, and $\mathbf{F}$, thermophoretic, diffusion-phoretic, and reactive forces, viscous resistance of the gaseous medium, and resultant of all the forces, $\mathrm{N} ; \mathrm{Kn}$ and Re, dimensionless Knudsen and Reynolds numbers; $\Psi(\mathbf{r})$ and $\mathbf{v}(\mathbf{r})$, stream function and vector velocity field in the gaseous medium, $\mathrm{m}^{3} / \mathrm{sec}$ and $\mathrm{m} / \mathrm{sec} ; p(\mathbf{r})$, pressure in the gas mixture, $\mathrm{Pa} ; C(\mathbf{r})$, scalar field of relative concentration of the first component of the gas mixture; $T(\mathbf{r})$ and $T^{\prime}(\mathbf{r})$, temperature
distributions outside the droplet and inside it, $\mathrm{K} ; T_{\mathrm{w}}$, average temperature on the aerosol-particle surface, $\mathrm{K} ; T_{0}$ and $C_{0}$, unperturbed values of the temperature and the relative concentration of the volatile component (at the location of the geometric center of the droplet in its absence), K and dimensionless; $\mathbf{A}_{T}=(\nabla T)_{\infty}$, constant gradient of temperature in the gas medium at infinity, $\mathrm{K} / \mathrm{m} ; K_{\mathrm{Tsl}}$ and $K_{\mathrm{Dsl}}$, dimensionless gaskinetic coefficients of thermal and diffusion slips of the binary gas mixture; $\left(V_{T T}, V_{T C}\right)$ and $\left(k_{T T}, k_{T C}\right)$, gaskinetic coefficients of temperature jumps, m and dimensionless; $D$, coefficient of mutual diffusion of the components of the gas mixture; $\mathrm{m}^{2} / \mathrm{sec} ; K_{\mathrm{TD}}$, dimensionless thermodiffusion relation; $\left(n_{1}, n_{2}\right),\left(m_{1}, m_{2}\right)$, and $\left(\lambda_{1}, \lambda_{2}\right)$, number concentrations, masses, and mean free paths of the gas molecules of the first and second sorts, $\mathrm{m}^{-3}, \mathrm{~kg}$, and $\mathrm{m} ; n^{\prime}$, number concentration of the molecules of the condensed phase, $\mathrm{m}^{-3} ; \rho$ and $\eta$, density and dynamic viscosity of the gas medium, $\mathrm{kg} / \mathrm{m}^{3}$ and $\mathrm{Pa} \cdot \mathrm{sec} ; \kappa$ and $\kappa^{\prime}$, coefficients of thermal conductivity of the gas medium and the droplet substance, $\mathrm{J} /(\mathrm{K} \cdot \mathrm{m}) ; p_{1 \mathrm{~s}}, n_{1 \mathrm{~s}}$, and $C_{\mathrm{s}}$, pressure and number and relative concentrations of the saturated vapor of the volatile condensed phase, $\mathrm{Pa}, \mathrm{m}^{-3}$, and dimensionless; $L$, specific vaporization heat, $\mathrm{J} / \mathrm{kg} ; \alpha$, dimensionless evaporation coefficient; $\mu$, molar mass of the droplet substance, $\mathrm{kg} / \mathrm{mole} ; k$ and $R_{\mathrm{g}}$, Boltzmann constant and universal gas constant, $\mathrm{J} / \mathrm{K}$ and $\mathrm{J} /(\mathrm{K} \cdot$ mole $) ; ~ \nabla$, vector differential operator of first order, $\mathrm{m}^{-1} ; \mathrm{E}^{2}$ and $\Delta$, differential operators of Stokes and Laplace of second order, $\mathrm{m}^{-2} ; \xi$, dimensionless angular variable; $C_{n}^{-1 / 2}(\xi)=J_{n}(\xi)$ and $C_{n}^{+1 / 2}(\xi)=P_{n}(\xi)$, dimensionless ultraspherical polynomials of Gegenbauer; $a, b, c, d, e, f$, $e^{\prime}, f^{\prime}, g$, and $h$, unknown dimensionless constants of expansion with a natural index. Subscripts: Tsl, thermal slip; Dsl, diffusion slip; Tph, thermophoretic; Dph, diffusion-phoretic; s, saturated vapor; TD, thermodiffusion; $\alpha$, reactive component; $\mathbf{v}$, vector velocity field; $T$, temperature; $T T$ and $T C$ at the temperature jumps, the first letter $T$ denotes the temperature field, the second letter denotes the conditionality of the temperature jump by the presence of the inhomogeneity in the temperature $T$ and the relative concentration $C$ respectively; 1 and 2 at the physical quantities denote molecules of the first and second sort; 0 , unperturbed values; $z$, projections onto the $O z$ axis; $s$, projection onto the tangential (s) direction; $n, r$, and $\theta$, projection onto the normal ( $\mathbf{n}$ ), radial ( $\mathbf{r}$ ), and tangential ( $\mathbf{i}_{\theta}$ ) directions; $\mathbf{w}$, wall; ', internal region; g, gas.

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